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DEPTH DEPENDENCE AND VERTICAL DIRECTIONALITY OF AMBIENT SEA NOI--ETC
AUG 79 W W RENNER, R C CAVANAGH N00014-79-C-0145

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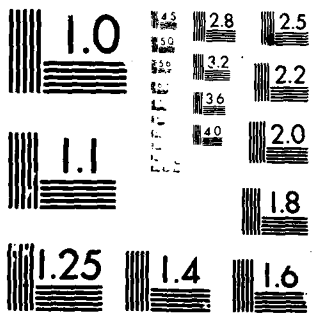

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DEPTH DEPENDENCE AND VERTICAL DIRECTIONALITY OF AMBIENT SEA NOISE

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DEPTH DEPENDENCE AND VERTICAL DIRECTIONALITY OF AMBIENT SEA NOISE

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Section 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

The performance of passive, low-frequency acoustic sensors is largely limited by prevailing ambient sea noise. In the past, the Navy has focused research on the omnidirectional (omni) spectrum levels and depth dependence of the noise field to optimize design and deployment of shallow and mid-water systems (e.g., sonobuoys). Because of recent developments in sonar arrays with vertical apertures, there is now considerable interest in the vertical directionality of ship and wind-generated noise. This report develops and applies a theoretical relationship connecting these two important properties of sea noise: depth dependence of the omni level and vertical directionality at a single depth.

The relationship is based on an analytical result which was documented a few years ago (Reference 1-1), but which has apparently not been exploited to date. It is somewhat limited by assumptions:

- the noise field is an "average" in time,
- geometric acoustics applies,
- the depth regime is limited to a single "sound channel" (see Subsection 2.1).

However, the result is not limited to range-independent environments and has applications to many problems, including:

- analysis and interpretation of noise data,
- insight into anomalies in the noise field (e.g., the "notch"),
- optimization of computer noise-prediction models.

The approach applies as well to range-averaged transmission loss (see Subsection 2-4). Specifically, under the assumptions listed, the relationship between range-averaged transmission loss as a function of depth and vertical arrival structure is similar to the relationship between the corresponding properties of sea noise discussed in this paper.

The present work was motivated by an effort (Reference 1-2) to improve the noise module of a vertical-array performance-prediction model used for Fleet support. The result allows omnidirectional depth-dependence data to give clues about the vertical directionality of the noise field. More detail on this background follows.

1.2 Background

A new generation of passive sonobuoys which employ vertical line arrays (VLA) can achieve significant gains in signal-to-noise ratio (compared to an omnidirectional sensor) by discriminating in favor of specific signal

arrival angles and against ambient noise arriving from other directions. A performance model for such a sensor requires a prediction of the vertical directionality of the ambient noise at the sonobuoy depth. The current Fleet-support model (ASRAP) estimates vertical directionality with an approach due to Talham (Reference 1-3) and extended from FANM (Reference 1-1). Details about the noise component of ASRAP can be found in Reference 1-4.

A problem with the current ASRAP noise module is the chronic prediction of a "notch" (absence of energy) at angles near the horizontal. Because of the assumption of a range-independent environment (i.e., sound speed and bathymetry do not change with range) and the use of geometric acoustics (i.e., no diffraction or scatter), the model will predict a null in the noise directionality whenever the sound speed at the receiver depth is less than the sound speed at some point above it. In contrast, models which account for range-dependence indicate that the notch is often partially filled and in many cases significantly filled.* Furthermore this is qualitatively supported by several noise vertical directionality measurements (Refs. 1-5 and 1-6).

Because of the limited quantity and quality of noise directionality data, additional information was sought

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* Diffraction and scatter are estimated to be of secondary importance at frequencies above 25 Hz.

to aid in determining the extent to which the notch is filled. The relationship discussed in this report allows ambient-noise depth-dependence measurements to be applied to the problem, and this report is chiefly concerned with how such data can be used to estimate vertical directionality.

1.3 Outline

The relationship between noise depth dependence and vertical directionality is considered in both directions. Section 2 discusses the assumptions and limitations of the approach, and then develops formulas and approximations for determining the depth-dependence from the vertical directionality at a specific depth. Section 3 then derives the depth-dependence function for several canonical directionality functions, and Section 5 provides examples of one of these depth-dependence functions for a variety of environments.

Section 4 deals with the inverse problem: determining the vertical directionality from the depth dependence. Two different approaches are considered: an analytic solution is developed for the case that the depth dependence has a particular representation; an error-minimization algorithm is applied when the vertical directionality has a particular form.

Section 2

NOISE MODEL, LIMITATIONS AND APPROXIMATIONS

This section gives relationships between omnidirectional ambient noise and vertical directionality, as functions of depth, based on a rather general model. The limitations and assumptions associated with the model are described, and approximate formulas derived.

2.1 The Model and Assumptions

Begin by assuming that the directionality of the noise field $N_S(\theta,\phi;z_0)$ is known. Here, N_S represents equivalent-plane-wave intensity, per steradian, at depth z_o, for vertical angle θ (measured from the horizontal) at azimuthal angle ϕ, in a small band of frequencies. Next suppose that geometric acoustics gives an accurate approximation of the field and that the medium is range-independent in a neighborhood of the receiver location (specifically, in a region including a ray's range-cycle; say, 35 miles at most). Azimuthal homogeneity is not required.

Under such assumptions, the vertical directionality (per steradian) can be viewed as the average in azimuth of arrivals:

$$N_S(\theta,z_o) \equiv \frac{1}{2\pi} \int_0^{2\pi} N_S(\theta,\phi;z_o)\, d\phi. \qquad (2\text{-}1)$$

Furthermore, since Snell's law holds locally, the noise reaching the receiver-depth z_0 at angle θ will reach depth z at angle $\hat{\theta}$,

$$\hat{\theta} = \text{sgn}(\theta)\cdot\arccos\left[\frac{c(z)}{c(z_0)}\cos\theta\right], \tag{2-2}$$

where c(z) is the sound speed at depth z. [If $\left|\frac{c(z)}{c(z_0)}\cos\theta\right| > 1$, then the path will not reach depth z].

Continue by assuming that the sources of noise are distributed as a continuum, homogeneous in range (radially from the receiver) over a ray's range cycle. For wind sources, this is reasonable. For ship sources, view the noise field as an average in time (and hence over source or receiver location). This additional premise allows arrivals to be translated in range, with the only error resulting from volume attenuation for the difference in path lengths (negligible at low frequency). Such translations, illustrated in Figure 2-1, lead to the important relationship,

$$N_s(\hat{\theta},\phi;z) \approx N_s(\theta,\phi;z_0) \tag{2-3}$$

and for vertical directionality (applying (2-1)):

$$N_s(\theta;z_0) \approx N_s(\hat{\theta};z) \tag{2-4}$$

where θ and $\hat{\theta}$ are related by (2-2).

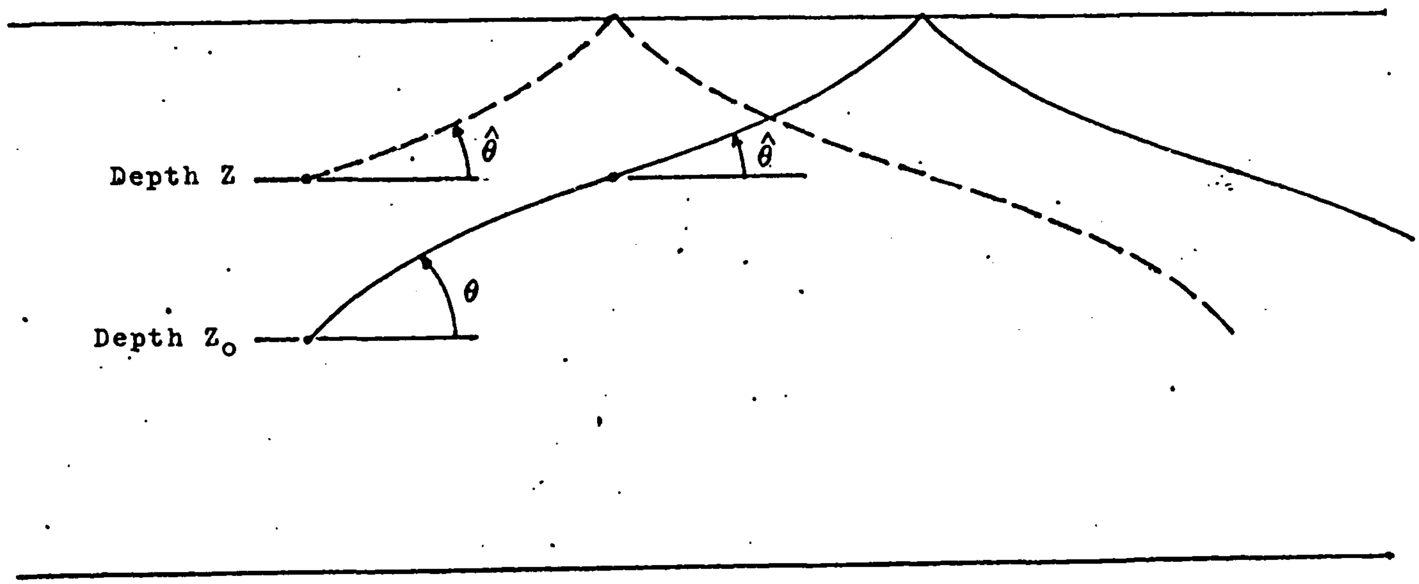


Figure 2-1 The translation of receiver depth z_o to z corresponds to the angle transformation of θ to $\hat{\theta}$.

The consequences of these observations are significant. For, once $N_s(\theta, \phi; z_o)$ is known for all θ at a given depth z_o, it can be estimated at any depth z, provided that every ray path that reaches z also reaches z_o. For the case that z_o is a local minimum in the sound speed profile, this requirement is met if z is in the same "sound channel" as z_o, i.e. there is no local maximum between z and z_o. Furthermore if the environment is assumed to be range independent, this restriction can be lessened to: no local maximum between z and z_o that is greater than the sound speed at the source depth (usually the surface). It is important to note that while all rays that reach depth z also reach z_o under these conditions, the converse is not true. This can be seen in that the transformation from is defined only on the domain $\alpha(z) < |\theta| \leq \frac{\pi}{2}$ where

$$\alpha(z) = \arccos \frac{c(z_o)}{c(z)} *.$$

As will be observed time and again in what follows, under the assumptions of the model the depth dependence of noise directionality is directly related to depth changes in sound speed. In fact, if z_1 and z_2 are in the same "sound channel" and if $c(z_1)=c(z_2)$, then the noise directionality $(N(\theta;z))$ is the same at z_1 and z_2. Thus, to get new information about the noise field, samples must be taken at depths with different sound speeds. To go one step further, a case of constant sound

* Note that $o < c(z_o)/c(z) \leq 1$. $\alpha(z)$ is taken as the value of the arccos between 0 and $\pi/2$.

speed should be viewed as degenerate; for then the directionality and omni levels are also constant with depth - so that directionality cannot be determined from depth dependence.

In summary, the noise vertical directionality depends on depth in a simple way (equations 2-3 and 2-4) under the assumptions that:

- Plane-wave geometric acoustics applies.

- Surface-image interference and diffraction effects are ignored (but see Subsection 2.3).

- Volume attenuation is small over 15 miles (low frequencies).

- The environment is range-independent within about 30 miles of the receiver location.

- Sources of noise are continuously distributed, and approximately homogeneous over ranges of 30 miles.

- The formulas are applied for depths within the same "sound channel".

Note that the assumptions about the environment and sources are equivalent to those of FANM II (Reference 2-1), and are consistent with any noise model which uses range-averaged transmission loss in a range-dependent environment (e.g., ASTRAL in an adiabatic case).

2.2 Omnidirectional Noise Versus Depth

Under the additional assumption that arrivals add incoherently, the omnidirectional noise at depth z_o can be calculated directly as:

$$N(z_o) = \iint_{\substack{\text{UNIT}\\ \text{SPHERE}}} N_s\,(\theta,\phi;z_o)\; d\Omega$$

$$= \int_0^{2\pi}\int_{-\pi/2}^{\pi/2} N_s(\theta,\phi;z_o)\,\cos\Theta\; d\theta\; d\phi. \qquad (2\text{-}5)$$

In terms of vertical directionality (see (2-1)),

$$N(z_o) = 2\pi\int_{-\pi/2}^{\pi/2} N_s(\theta;z_o)\,\cos\Theta\; d\Theta. \qquad (2\text{-}6)$$

(Again, notice that if c(z) is constant, then so are $N_s(\theta;z)$ and N(z) as functions of z).

When $c(z_o)$ is a local minimum in the sound speed profile and z is in the same sound channel, then the omnidirectional depth dependence can be calculated from:

$$N(z) = 2\pi\int_{-\pi/2}^{\pi/2} N_s(\hat\theta;\, z)\,\cos\hat\theta\; d\hat\theta$$

$$\approx 2\pi\int_{-\pi/2}^{\pi/2} N_s(\theta;\, z_o)\,\cos\hat\theta\; d\hat\theta \qquad (2\text{-}7)$$

$$= 2\pi\left[\int_{\alpha(z)}^{\pi/2} - \int_{-\pi/2}^{-\alpha(z)}\right] d\theta N_s(\theta;\, z_o)F(\theta,\, z),$$

where

$$F(\theta,z) = \frac{\sin\theta \cos\theta}{\cos\left[\alpha(z)\right] \sqrt{\cos^2\left[\alpha(z)\right] - \cos^2\theta}} \qquad (2\text{-}8)$$

and

$$\alpha(z) = \arccos\left[\frac{c(z_o)}{c(z)}\right].$$

In the case that $N_s(\theta;z_o)$ is symmetric about the horizontal, (2-7) simplifies to

$$N(z) = 4\pi \int_{\alpha(z)}^{\pi/2} N_s(\theta,z_o)F(\theta,z)d\theta. \qquad (2\text{-}9)$$

Examples of the application and interpretation of these results are found in Section 3.

2.3 Surface-Image Interference and Diffraction Effects

To this point, the changes in vertical directionality with receiver depth have been attributed completely to the geometric change in ray-path structure, calculated directly from Snell's law in a neighborhood of the receiver. Moreover, in the last subsection a formula for the depth dependence of the total (omnidirectional) level was derived under the additional assumption that the noise consists of plane-wave arrivals which add on a random-phase basis (incoherently). The following discussion is concerned with two mechanisms which can modify the results obtained from the simplified ray model: surface-image interference (SII) and diffraction.

Concentrate first on noise arrivals from a single, distant point source. If these arrivals were added in phase, a complex multipath-interference field would be observed as the source or receiver moved in range or depth. Because of the assumptions of time- or range-averaged noise (as reflected in the noise-source distribution) associated with the model of this paper, only those fluctuations which persist in range need be considered; the others are smoothed in the averaging process. Following the rationale of FACT (Ref 2-2), we limit the candidate interfering paths to pairs of long-range arrivals which differ in their history by a single surface reflection near the receiver. The resulting effect can substantially increase low-frequency transmission loss for receivers near the surface, and is usually called "Surface-Image Interference" (in special cases, "Lloyd's Mirror").

To account for the SII Mechanism, first note that the noise directionality is unaltered; phases can be assigned to the arrivals, but until the arrivals are summed there is no manifestation of the interference (There is an implicit assumption here that arrivals from two "sources" add incoherently). Focus than on the calculation of the omni level (N(z)). At least two approaches make sense:

- Arrivals from above and below with the same (absolute-value) angle are presumed to interfere. The well-known approximation can be used to modify the sum:

  $$I_2 = 4 I_1 \sin^2 \left(\frac{\omega z \sin\theta}{c}\right) \quad , \qquad (2\text{-}10)$$

  where I_1 is the intensity for a single arrival at angle θ, ω is radial frequency, c is sound speed, and z is receiver depth. More precise formulas including the influence of the local sound-speed variations with depth are available (see, e.g., Ref. 2-3). When the changes of I_2 with depth become rapid (as ω, z, or θ increase), I_2 is replaced by the incoherent (RMS) sum.

- The TAPPS noise module (Ref. 2-4) predicts noise depth dependence for towed array systems and attributes all variation to SII. Since the array is presumed shallow, the frequency low, and arrivals from ships at small angles, such a treatment is not unreasonable. Note, however, that the sound speed

profile at the array is used only to calculate limiting ray arrivals from the surface and bottom.

TAPPS considers two regimes of noise arrivals: paths which do not suffer bottom attenuation, and those which do. In each case, an average of the sum of (2-10) over the appropriate angular aperture is used to find omni noise depth dependence. Only in the bottom-bounce case are the individual arrivals within the aperture presumed to change in intensity with angle. The TAPPS algorithm is nearly the same as that of the FACT shallow-water routine (Ref. 2-1).

Either of these approaches can be applied directly to the model described in Subsection 2.2 to account for SII. It should be emphasized that the effect is important only for small values of $\frac{\omega z \sin\theta}{c}$.

Other physical mechanisms not included in the model are those related to "wave" or diffraction phenomena. They can be important at low frequencies, but are not predicted by geometric acoustics. For example:

- The noise field actually extends into "shadow" regions bounding the "sound channel." The intensity and extent of the diffracted field are inversely proportional to frequency and sound-speed gradient.

- Ray arrivals which cycle within a constrained depth interval (e.g., in a surface duct or about a channel axis) are subject to model attenuation as frequency decreases (low-frequency cut off).

Such effects can be included in a geometric-acoustic framework, as has been done in the ASTRAL (Ref. 2-5) and other transmission-loss models.

In summary, SII and diffraction can significantly modify noise depth dependence at low frequency. There are straightforward methods for estimating the contribution of each. The remainder of this paper will concentrate on the geometric aspects introduced in Subsection 2.1 without further consideration of these other phenomena.

2.4 Application to Transmission Loss

The results of this paper apply almost without change to range-averaged transmission loss. Specifically, suppose $S(\theta;z_Q)$ represents transmission arrival structure at the receiver depth z_Q, averaged over an interval of source range exceeding the longest ray period (say, 35 nm). Make assumptions similar to those listed in Subsection 2.1:

- Plane-wave, geometric acoustics applies.

- SII and diffraction are ignored.

- Volume attenuation is small over 15 nm.

- The "source" or receiver is distributed over 35 nm (i.e., TL is averaged over that range).

- Formulas, are applied for depths at the receiver within the same "sound channel".

- The environment is range-independent within about 35 nm of the receiver.

In that case, formula (2.4) applies to :

$$S(\theta;z_o) = S(\hat{\theta};z)$$

for $$\hat{\theta} = \mathrm{sgn}(\theta) \cdot \arccos\left(\frac{c(z)}{c(z_o)}\cos\theta\right).$$

Furthermore, the "total" (as received by an omnidirectional receiver) sound intensity $S(z)$ satisfies the analogy of (2-6):

$$S(z)=\int_{-\pi/2}^{\pi/2} S(\theta;\ zo)\ d\hat{\theta}.$$

The only real difference between the transmission and noise problem is in the dimension of the sound field.

The formulas can be applied as in the case of noise to relate arrival structure and depth dependence. The last of the listed assumptions should be emphasized:

it precludes application of the result to receivers located in locally range-dependent environments (e.g., it could not be applied to the ASTRAL problem with receiver over a sloping bottom).

Finally, there is also an obvious application to TL and AN computer models, already known but not always recognized. In practice, rays are usually traced from receiver to source. If the receiver is placed at the channel axis, and the individual ray intensities stored, then the formulas for angle transformation allow the arrival structure or directionality to be constructed at other depths within the same "sound channel" without additional ray tracing. The only error suffered is in volume attenuation over at most one ray cycle.

Section 3

EXAMPLES OF DEPTH DEPENDENCE FOR CANONICAL DIRECTIONALITY

For certain realistic, albeit simplistic, vertical directionality functions, the noise depth-dependence function can be found analytically. This section derives the depth dependence functions for three such cases. In all cases $c(z_o)$ is a local minimum in the sound speed profile and z is restricted to the same sound channel as z_o (i.e., no local maximum between z_o and z).

3.1 Isotropic Vertical Directionality

For frequencies at which the wind noise dominates, some measurements show little vertical directionality in the ambient noise. Hence consider the case where,

$$N_s(\theta; z_o) = N, \ -\pi/2 \leq \theta \leq \pi/2.$$

Then the omnidirectional noise intensity at z_o is,

$$\begin{aligned} N(z_o) &= 2\pi \int_{-\pi/2}^{\pi/2} N_s(\theta; z_o) \cos\theta \, d\theta \\ &= 4\pi N, \end{aligned}$$

and at z,

$$N(z) = 4\pi \int_{\alpha(z)}^{\pi/2} N_s(\theta; z_o) \cdot F(\theta, z) d\theta$$

$$= 4\pi N \int_{\alpha(z)}^{\pi/2} F(\theta, z) d\theta$$

$$= 4\pi N \left[\frac{\sqrt{\cos^2 [\alpha(z)] - \cos^2 \theta}}{\cos [\alpha(z)]} \right]_{\theta=\alpha(z)}^{\pi/2} = 4\pi N.$$

Thus an isotropic vertical noise pattern yields a constant depth-dependence function.

3.2 Vertical Directionality as a Step Function

This example illustrates the "notch" problem mentioned earlier. Let c_s be the sound speed at the surface and c_B be the sound speed at the bottom with

$$c(z_o) < c_s \leq c_B$$

$$\theta_s = \arccos \left[\frac{c(z_o)}{c_s} \right]$$

$$\theta_B = \arccos \left[\frac{c(z_o)}{c_B} \right] \qquad (3-2)$$

Noise models which place the source at the surface and presume range-independent environments will predict that $N_s(\theta, z_o)$ is greatest when $\theta_s \leq |\theta| < \theta_B$ and near zero when $0 \leq |\theta| < \theta_s$. Consider then the case (see Fig. 3-1) for which,

$$N_s(\theta, z_o) = \begin{cases} N_1, 0 \leq |\theta| < \theta_s \\ N_2, \theta_s \leq |\theta| < \theta_B \\ N_3, \theta_B \leq |\theta| \leq \pi/2 \end{cases}.$$

Then,

$$N(z_o) = 2\pi \int_{-\pi/2}^{\pi/2} N_s(\theta; z_o) \cos\theta d\theta$$

$$= 4\pi \left[N_1 \int_0^{\theta_s} \cos\theta d\theta + N_2 \int_{\theta_s}^{\theta_B} \cos\theta d\theta + N_3 \int_{\theta_B}^{\pi/2} \cos\theta d\theta \right]$$

$$= 4\pi \left[N_1 \sin\theta_s + N_2 (\sin\theta_B - \sin\theta_s) + N_3 (1 - \sin\theta_B) \right] . \quad (3\text{-}3)$$

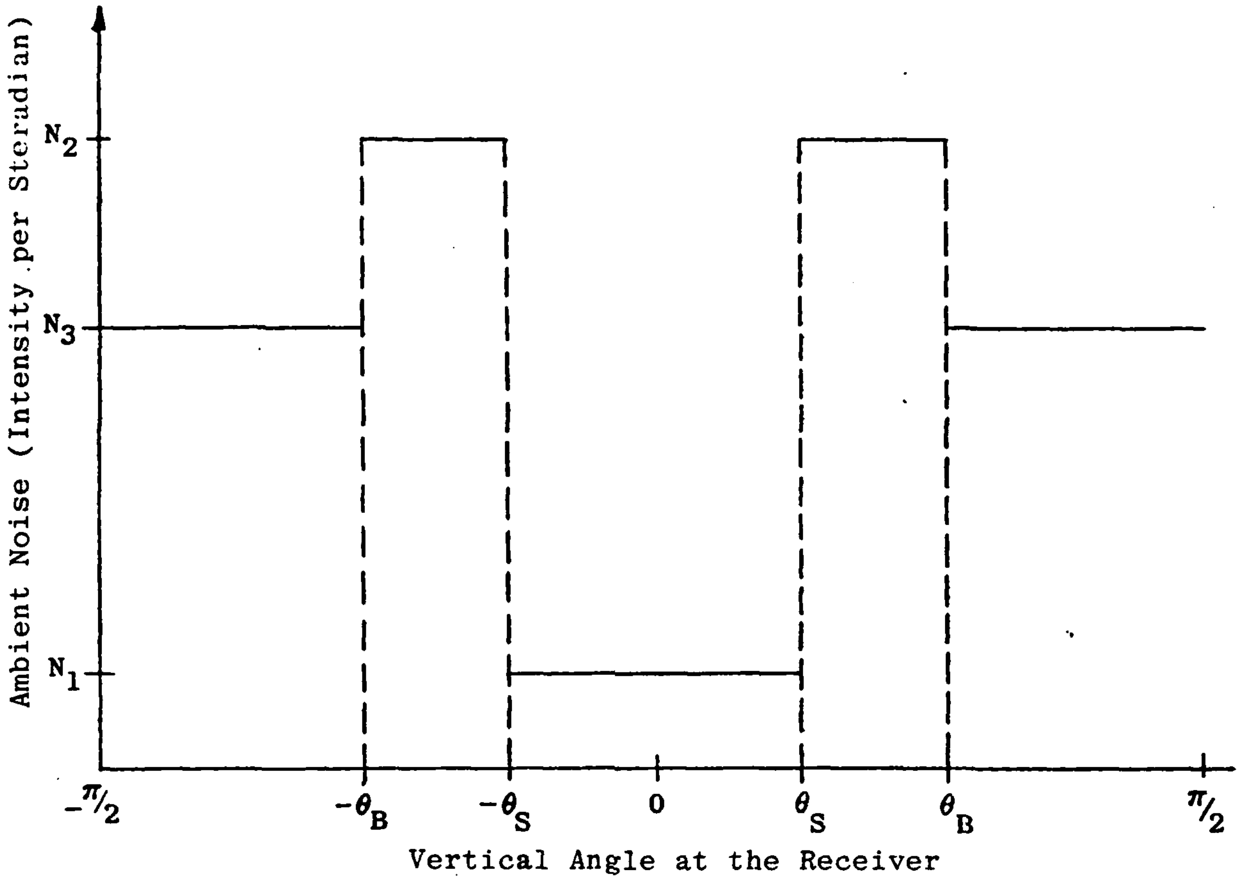


Figure 3-1

Vertical Directionality as a Step Function

3.3 <u>Vertical Directionality with a Singularity</u>

Let c_s denote the sound speed at the surface, $\theta_s = \arccos\left(\frac{c(z_o)}{c_s}\right)$, and $\theta_G(\theta) = \arccos\left[\frac{c_s}{c(z_o)} \cos\theta\right]$. In Talham-type models (e.g., FANM or ASRAP References 1-1 and 1-4), an isotropic source directivity results in:

$$N_s(\theta;z_o) \propto \frac{1}{\sin\theta_G(\theta)} \text{ as } \theta \rightarrow \theta_s$$

so that

$$\lim_{\theta\rightarrow\theta_s} N_s(\theta;z_o) = \infty .$$

Now

$$\frac{1}{\sin\theta_G} = \frac{\cos\theta_s}{\sqrt{\cos^2\theta_s - \cos^2\theta}} ,$$

so consider the noise directionality function (Fig. 3-2)

$$N_s(\theta;z_o) = \begin{cases} N_1 & , \; 0 \leq |\theta| < \theta_s \\ N_2 \cos\theta_s / \sqrt{\cos^2\theta_s - \cos^2\theta} & , \; \theta_s \leq |\theta| < \theta_B \\ N_3 & , \; \theta_B \leq |\theta| \leq \frac{\pi}{2} \end{cases}$$

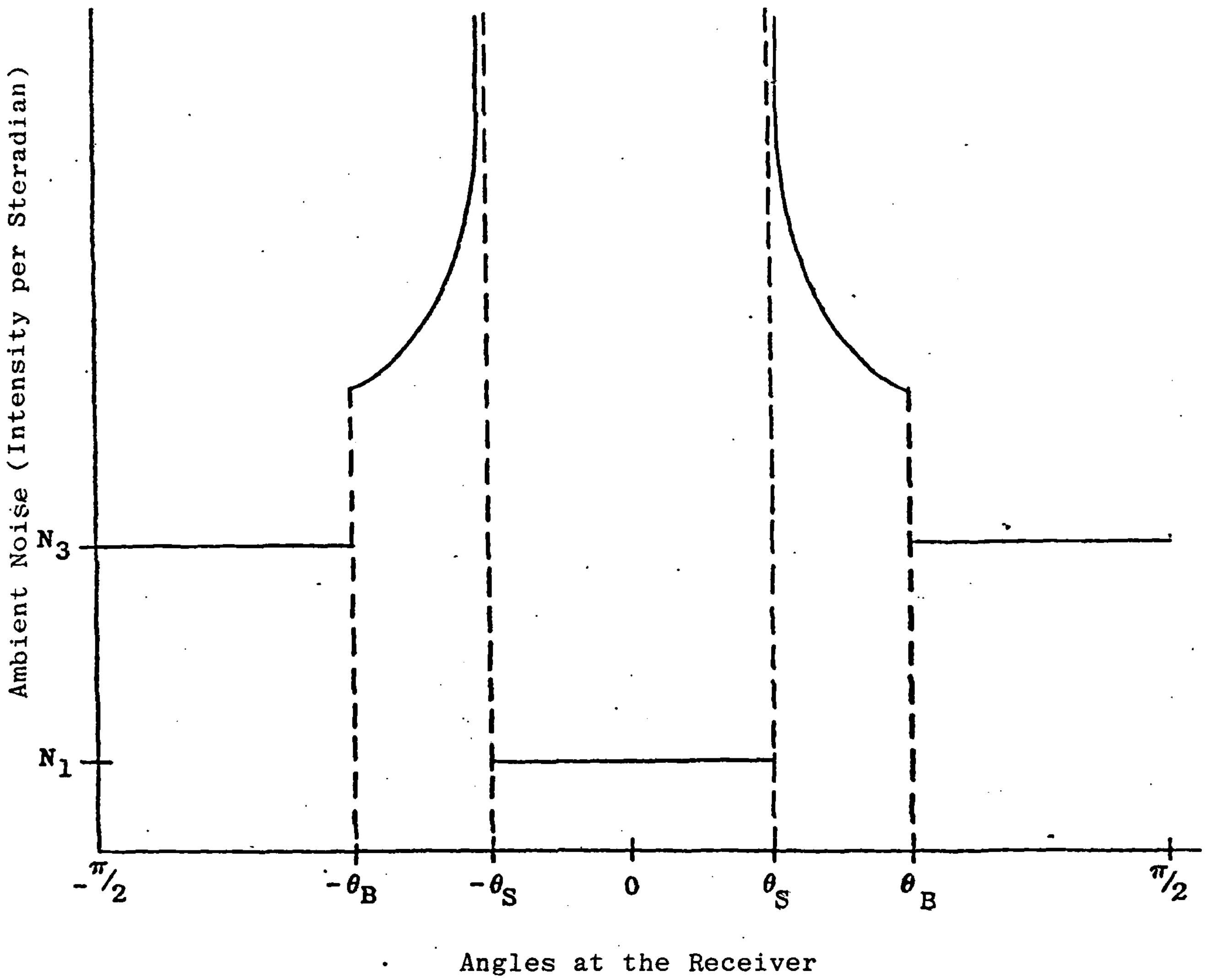


Figure 3-2

Vertical Directionality with a Singularity

Then

$$N(z) = N_1 \sin\hat{\theta}_s + \frac{N_2 \cos\theta_s}{\cos\alpha(z)} \int_{\theta_s}^{\theta_B} \frac{\sin\theta\cos\theta d\theta}{(\cos^2\theta_s - \cos^2\theta)^{\frac{1}{2}}(\cos^2\alpha(z) - \cos^2\theta)^{\frac{1}{2}}} + N_3(1-\sin\hat{\theta}_B)$$

where $\hat{\theta}_s$ and $\hat{\theta}_B$ are as defined in (3-2).

Letting $\chi = \cos^2\theta$, the integral above becomes

$$-\frac{1}{2}\int_{\cos^2\theta_s}^{\cos^2\theta_B} \frac{d\chi}{(\cos^2\theta_s - \chi)^{\frac{1}{2}}(\cos^2\alpha(z) - \chi)^{\frac{1}{2}}}$$

$$= -\frac{1}{2}\ln\left[2(\cos^2\theta_s - \chi)^{\frac{1}{2}}(\cos^2\alpha(z) - \chi)^{\frac{1}{2}} + 2\chi - \cos^2\theta_s - \cos^2\alpha(z)\right]\Bigg|_{\chi=\cos^2\theta_s}^{\cos^2\theta_B}$$

$$= \frac{1}{2}\ln\left[\frac{\cos^2\theta_s - \cos^2\alpha(z)}{2(\cos^2\theta_s - \cos^2\theta_B)^{\frac{1}{2}}(\cos^2\alpha(z) - \cos^2\theta_B)^{\frac{1}{2}} + 2\cos^2\theta_B - \cos^2\theta_s - \cos^2\alpha(z)}\right] \quad (3\text{-}4)$$

*See Reference 3-1

Now for $c(z) \neq c_s$, $N(z)$ is finite. However as $c(z) \rightarrow c_s$ then $\cos\alpha(z) \rightarrow \cos\theta_s$ and thus the expression in (3-4) approaches infinity. The reason for the singularity is discussed in Reference 1-1; it is a result of the fact that range-averaged transmission loss calculated with ray acoustics, has a finite average at smooth caustics but not at horizontal, cusped caustics. The latter occur when source and receiver are at depths with the same sound speed.

Section 4

INVERSE PROBLEM

To this point the discussion has focused on how the ambient noise depth dependence can be approximated from the noise directionality at a fixed depth. Of equal importance is the inverse problem: When can the directionality be determined from the depth dependence? This section deals with two approaches, the first an analytic solution for depth-dependence functions of a particular form and the second an error minimization algorithm for directionality functions similar to those of subsection 3.2.

4.1 Analytic Solution

CASE 1 Symmetry in θ

Assume $N_s(\theta, z_o)$ is symmetric in θ for z_o fixed at the axis (minimum sound speed) of a "sound channel." Then recalling (2-9)

$$N(z) = 4\pi \int_{\alpha(z)}^{\frac{\pi}{2}} N_s(\theta, z_o) \frac{\sin\theta \cos\theta}{\cos\alpha(z) \sqrt{\cos^2\alpha(z) - \cos^2\theta}} d\theta \cdot \quad (4\text{-}1)$$

where $\alpha(z) = \arccos\left(\frac{c(z_o)}{c(z)}\right)$, with $0 \leq \alpha(z) \leq \pi/2$.

This relationship can be viewed as a mapping of vertical directionality functions $\{N_s(\theta; z_o)\}$ into omni depth functions $\{N(z)\}$. An analytic solution to the inverse mapping (from $\{N(z)\}$ to $\{N_s(\theta; z_o)\}$) is sought.

Recall (Section 2) that whenever $c(z_1) = c(z_2)$ and z_1, z_2 are in the same "sound channel", then

$$N_s(\theta;\ z_1) = N_s(\theta;\ z_2),\ \text{for all } \theta,$$

and

$$N(z_1) = N(z_2).$$

It makes sense then to define c^{-1} as a multivalued function, and

$$N(c^{-1}(c(z))) = N(\hat{z}),$$

where $z=\hat{z}$ or any depth with $c(z) = c(\hat{z})$. In particular,

$$N\left(c^{-1}\left(\frac{c(z_o)}{\cos\ \alpha(z)}\right)\right) = N(z)$$

and equation (4-1) can be written in the form of an Abel integral equation (per Reference 4-1):

$$\frac{1}{4\pi} N\left(c^{-1}\left[\frac{c(z_o)}{\cos\alpha(z)}\right]\right)\cos\alpha(z) = \int_{\alpha(z)}^{\frac{\pi}{2}} N_s(\theta;z_o)\frac{\sin\theta\ \cos\theta}{\sqrt{-\cos^2\theta - (-\cos^2\alpha(z))}}\,d\theta. \tag{4-1A}$$

Notice that $\alpha(z)$ is the independent variable (not z), and that the equation is trivial if $\alpha(z)$ is constant. In fact, assuming continuity of $c(z)$, if $\alpha(z)$

were independent of z, it would have value $\alpha(z) \equiv 0$ and $\cos\alpha(z) \equiv 1$. In that case, equation (4-1) reduces to

$$\frac{1}{4\pi} N_o \equiv \frac{1}{4\pi} N(z\ ) = \int_0^{\pi/2} N_s(\theta;\, z_o)\cos\theta d\theta, \text{ for all } z,$$

and <u>any</u> directionality function $N_s(\theta;z_o)$, suitably normalized, solves the integral equation. Again, constant sound speed leads to a degenerate relationship.

The integral equation is non-trivial as long as $\alpha(z)$ is non-constant. If $\alpha(z)$ is continuous on some interval in depth and if N and c are suitably smooth, then the Abel equation has the unique solution (in the class of continuously differentiable functions; Ref 4-1):

$$N(\theta;z_o)\sin\theta\cos\theta = -\frac{1}{2\pi^2}\frac{d}{d\theta}\int_\theta^{\frac{\pi}{2}} \frac{N\left(c^{-1}\left[\frac{c(z_o)}{\cos u}\right]\right)\cos^2 u \sin u}{\sqrt{-\cos^2 u - (-\cos^2\theta)}}\,du.$$

For v = cos u, this simplifies to

$$N_s(\theta;z_o) = \frac{1}{2\pi^2\sin\theta\cos\theta}\frac{d}{d\theta}\int_{\cos\theta}^{0} N\left(c^{-1}\left[\frac{c(z_o)}{v}\right]\right)\frac{v^2\,dv}{\sqrt{\cos^2\theta - v^2}}$$

The directionality $N_s(\theta;z_o)$ can thus be determined explicitly from the depth-dependence function N(z) via the latter formula. Numerical quadrature and differentiation are usually required. There are, however, special forms

for N(z) in terms of c(z) which allow for closed-form solutions. Suppose, for example, that c(z) is not constant and that N(z) can be written as:

$$N(z) = A_o + A_1\left[\frac{c(z_o)}{c(z)}\right] + \cdots + A_n\left[\frac{c(z_o)}{c(z)}\right]^n$$

Then

$$N\left(c^{-1}\left[\frac{c(z_o)}{v}\right]\right) = A_o + A_1 v + \cdots + A_n v^n$$

and thus

$$N_s(\theta;z_o) = \frac{1}{2\pi^2 \sin\theta\cos\theta}\frac{d}{d\theta}\left[\int_{\cos\theta}^{o}\left(A_o+A_1v+\cdots+A_nv^n\right)\frac{v^2dv}{\sqrt{\cos^2\theta-v^2}}\right]$$

$$= \frac{1}{2\pi^2\sin\theta\cos\theta}\frac{d}{d\theta}\left[-\sum_{m=1}^{\left[\frac{n}{2}\right]+1}\frac{(2m)!\pi A_{2(m-1)}}{(m!)^2\cdot 2^{2m+1}}\cos^{2m}\theta - \sum_{m=1}^{\left[\frac{n+1}{2}\right]}\frac{2^{2m}(m!)^2A_{2m-1}}{(2m+1)!}\cos^{2m+1}\theta\right]^{*}$$

$$= \frac{1}{2\pi^2}\left[\sum_{m=1}^{\left[\frac{n}{2}\right]+1}\frac{(2m)!\pi A_{2(m-1)}}{\left[(m-1)!\right]^2\cdot 2^{2m}}\cos^{2(m-1)}\theta + \sum_{m=1}^{\left[\frac{n+1}{2}\right]}\frac{2^{2m}(m!)^2 A_{2m-1}}{(2m-1)!}\cos^{2m-1}\theta\right]. \quad (4\text{-}2)$$

---

* See Ref. 4-2

In particular, for n=0 (i.e., depth-dependence function is constant), the noise field must be isotropic:

$$N_s(\theta;z_o) = \frac{a_o}{4\pi} \quad .$$

Case 2 Asymmetry in θ

Suppose $N_s(\theta;z_o)$ is not symmetric. Define

$$g(\theta) = \frac{N_s(\theta;z_o)}{N_s(-\theta;z_o)} \text{ for } 0 \leq \theta \leq \pi/2$$

and

$$\overline{N}_s(\theta;z_o) = \frac{1}{2}\left[N_s(\theta;z_o)+N_s(-\theta;z_o)\right]$$
$$=\frac{1}{2}N_s(\theta;z_o)\left[1+g(\theta)\right], \text{ for } 0 \leq \theta \leq \pi/2.$$

Then, recalling (2-7)

$$N(z) = 2\pi\left[\int_{\alpha(z)}^{\pi/2} - \int_{-\pi/2}^{-\alpha(z)}\right] d\theta N_s(\theta;z_o)\cdot F(\theta,z)$$

$$= 2\pi\int_{\alpha(z)}^{\pi/2} N_s(\theta;z_o)F(\theta,z)d\theta+2\pi\int_{\alpha(z)}^{\pi/2} g(\theta)N_s(\theta;z_o)\cdot F(\theta,z)d\theta ,$$

and so,

$$N(z) = 4\pi \int_{\alpha(z)}^{\pi/2} \overline{N}_s(\theta;z_o)F(\theta,z)d\theta.$$

A solution for $\overline{N}_s(\theta;z_o)$ can be found as before in (4-2). Finally, $g(\theta)$ can be estimated by the sum of the volume attenuation along the additional ray-path length and the bottom loss/attenuation, if any, for the additional bottom bounce (see Figure 4-1).

4.2 Error Minimization Algorithms

As an alternate to the explicit inverse approach, consider an approximate method whereby $N_s(\theta;z_o)$ is assumed to have a special form, depending on several parameters. The resulting N(z) calculated from (4-1) is compared to the actual depth function, and the parameters adjusted to minimize the error. In the illustration below, $N_s(\theta;z_o)$ is assumed to depend on 3 parameters, but the approach is directly generalizable to more complicated forms. An intuitively satisfying observation is made here: the problem is well defined as long as the number of distinct samples (i.e., those at distinct sound speeds) of N(z) is at least as great as the number of unknown parameters defining $N_s(\theta;z_o)$.

Let $\{z_i\}_{i=1}^{m}$ be a sequence of depths all contained in the same sound channel as z_o with $c(z_i) > c(z_o)$ for all i.

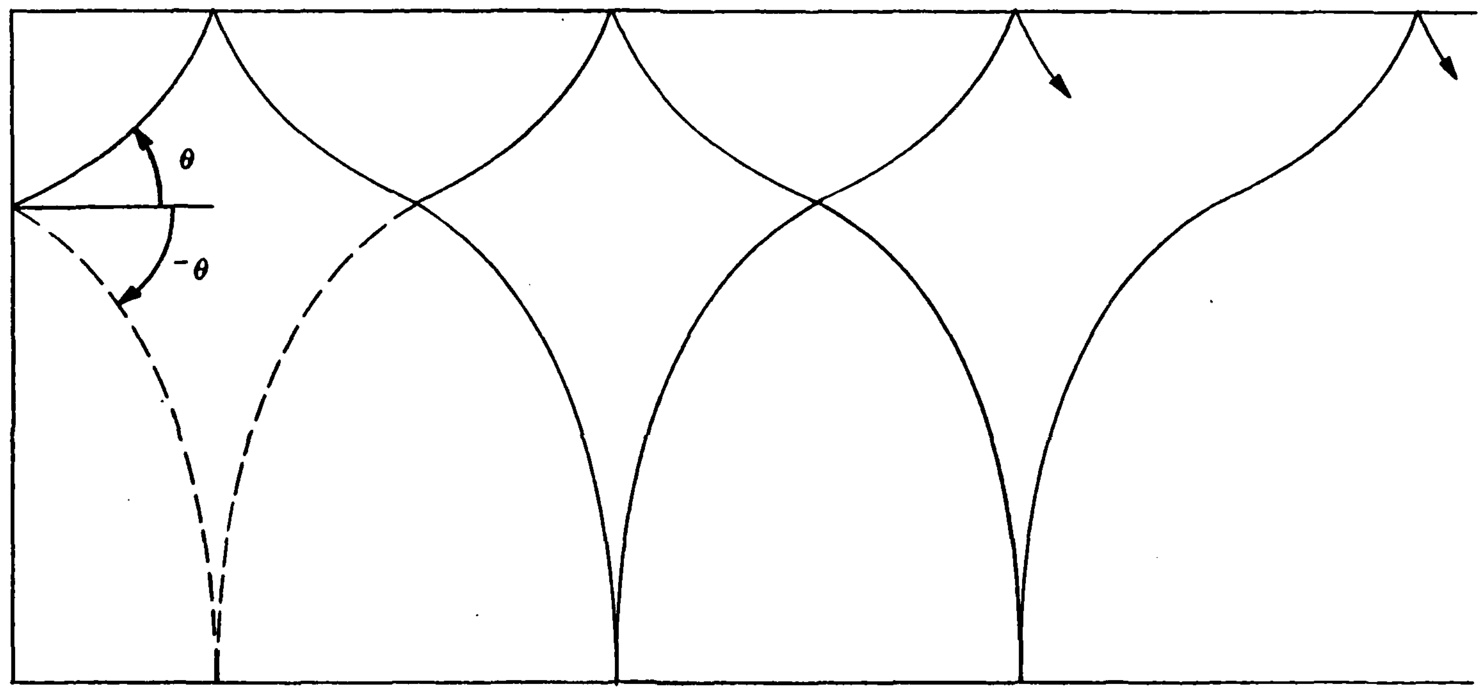


Figure 4-1. $g(\theta)$ is approximated by additional attenuation along dashed ray path.

Let $N(z_i)$ denote the omnidirectional noise level at depth z_i. As was shown earlier, if the noise directionality at z_o is

$$N_s(\theta;z_o) = \begin{cases} N_1, & |\theta| < \theta_s \\ N_2, & \theta_s \le |\theta| < \theta_B \\ N_3, & \theta_B \le |\theta| \le \pi/2 \end{cases} , \tag{4-3}$$

then

$$N(z_i) = 4\pi \left[N_1\alpha_1 + N_2(\beta_i - \alpha_i) + N_3(1-\beta_i) \right] \tag{4-4}$$

where

$$\alpha_i = \sin\left[\cos^{-1}\left(\frac{c(z_i)}{c(z_o)}\cos\theta_s\right)\right] = \frac{\sqrt{c_s^{\,2} - c(z_i)^2}}{c_s}$$

$$\beta_i = \sin\left[\cos^{-1}\left(\frac{c(z_i)}{c(z_o)}\cos\theta_B\right)\right] = \frac{\sqrt{c_B^{\,2} - c(z_i)^2}}{c_B}$$

Thus, given $\{N(z_i)\}$ and assuming N_s takes form (4-3), we wish to find non-negative numbers (N_1, N_2, N_3) such that (4-4) is true for all i.

First note that when only three depths are given, with $c(z_i) \neq c(z_j)$ for $i \neq j$, then ignoring any error terms, the problem is simply to solve a system of three linear equations in three unknowns. It can be shown that the matrix of coefficients,

$$M = \begin{pmatrix} \alpha_1 & \beta_1-\alpha_1 & 1-\beta_1 \\ \alpha_2 & \beta_2-\alpha_2 & 1-\beta_2 \\ \alpha_3 & \beta_3-\alpha_3 & 1-\beta_3 \end{pmatrix}$$

is non-singular under the conditions that the $c(z_i)$ are distinct and that $c_S \neq c_B$. Hence there exists a unique solution, namely

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = M^{-1} \begin{pmatrix} N(Z_1) \\ N(Z_2) \\ N(Z_3) \end{pmatrix}$$

Now when more than three depths are given, the system of equations is overdetermined. In practical applications, equality in (4-4) for all i is virtually impossible because of the special form of (4-3) and the measurement accuracy of $N(z_i)$. Therefore, some sort of error minimization process must be used. Two such processes were considered: (1) a least-squares estimate and (2) a linear program. Of the two the least-squares estimate requires fewer computer resources. The formulations of these two problems are given in the following subsections.

4.2.1 Least-Squares Estimate

Problem: Find the least-squares estimators n_1, n_2, n_3, of N_1, N_2, N_3, for function

$$N(z) = 4\pi \left[N_1 \alpha + N_2 (\beta-\alpha) + N_3 (1-\beta) \right]$$

given data points $(\alpha_i, \beta_i, N(z_i), i = 1, 2, \ldots, m)$

Solution: The sum of the squared deviations between the measured and the theoretical values of $N(z_i)$ is

$$S = \sum_{j=1}^{m} \left\{ N(z_i) - 4\pi \left[N_1 \alpha_i + N_2 (\beta_i - \alpha_i) + N_3 (1-\beta_i) \right] \right\}^2.$$

Then setting the partial derivatives $\frac{\partial S}{\partial N_1}$, $\frac{\partial S}{\partial N_2}$ and $\frac{\partial S}{\partial N_3}$ to zero, we obtain n_1, n_2, and n_3 (the least-squares estimators of N_1, N_2 and N_3) as solutions of the equations

$$\sum N(z_i)\alpha_i = 4\pi \left[N_1 \sum \alpha_i^2 + N_2 \sum \alpha_i(\beta_i - \alpha_i) + N_3 \sum \alpha_i (1-\beta_i) \right]$$

$$\sum N(z_i)(\beta_i - \alpha_i) = 4\pi \left[N_1 \sum \alpha_i(\beta_i - \alpha_i) + N_2 \sum (\beta_i - \alpha_i)^2 + N_3 \sum (\beta_i - \alpha_i)(1-\beta_i) \right]$$

$$\sum N(z_i)(1-\beta_i) = 4\pi \left[N_1 \sum \alpha_i (1-\beta_i) + N_2 \sum (\beta_i - \alpha_i)(1-\beta_i) + N_3 \sum (1-\beta_i)^2 \right].$$

4.2.2 Linear Program

Problem: Define y_i to be the absolute difference between the measured noise intensity at the depth z_i and the theoretical noise intensity as given by (4-4). For each i assign a relative cost (penalty), c_i, to be applied to the y_i. If the objective is to minimize the total cost, then the problem can be solved by the linear program:

Minimize: $$\sum_{i=1}^{m} c_i y_i ,$$

Subject to:

$$\left| N(z_i) - 4\pi \left[N_1 \alpha_i + N_2(\beta_i - \alpha_i) + N_3(1-\beta_i) \right] \right| \leq y_i, i=1,\cdots,m$$

$$N_j \geq 0 \ , \ j=1,2,3$$

$$y_i \geq 0 \ , \ i=1,2,\cdots,m \ .$$

The linear program is advantageous in that a variety of meaningful options for the cost function can be used (e.g., minimize: max $\{c_i \cdot y_i\}$). Additionally, constraints that reflect a priori knowledge can easily be included (e.g., $N_2 \geq 2N_3$).

Section 5
EXAMPLES OF EFFECTS OF NOTCH FILLING ON DEPTH DEPENDENCE

In subsection 3-2, it was shown that when the vertical directionality at the sound-speed minimum ($c(z_o)$) takes special form

$$N_s(\theta;z_o) = \begin{cases} N_1 & , \; 0 \leq |\theta| < \theta_s \\ N_2 & , \; \theta_s \leq |\theta| < \theta_B \\ N_3 & , \; \theta_B \leq |\theta| < \frac{\pi}{2} \end{cases} ,$$

then the depth dependence function is given by

$$N(z) = 4\pi \left[N_1 \sin\hat{\theta}_s + N_2(\sin\hat{\theta}_B - \sin\hat{\theta}_s) + N_3(1-\sin\hat{\theta}_B) \right] ,$$

where

$$\hat{\theta}_s = \arccos \left[\frac{c(z)}{c(z_o)} \quad \cos\theta_s \right]$$
$$\hat{\theta}_B = \arccos \left[\frac{c(z)}{c(z_o)} \quad \cos\theta_B \right] .$$

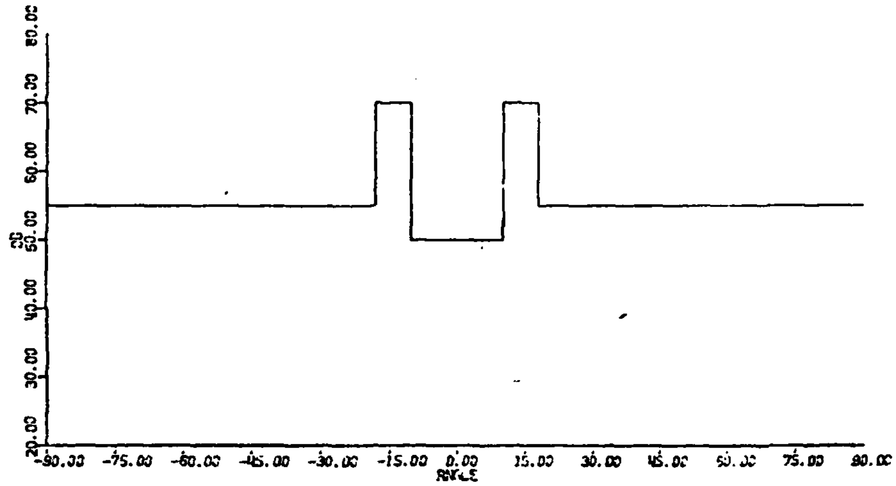
This section provides examples of depth-dependence functions in a variety of environments for vertical-directionality functions as defined above. Furthermore these examples illustrate the relative effect of "notch filling" in these environments.

Figures 5-1 thru 5-4 are for a typical deep-water sound-speed profile with a well-defined sound channel. At the channel axis the surface and bottom grazing angles are approximately 10.3^{o} and 18^{o} respectively.

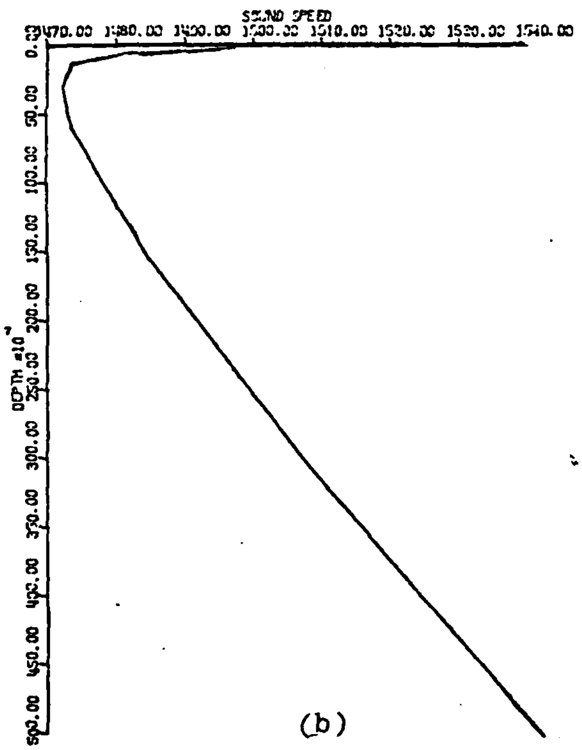
In Figure 5-1c the depth dependence varies by about 2.5 dB, reaching a minimum at the axis of the sound channel and a maximum at the surface and the conjugate depth. This variation, which occurs despite the assumption that the levels N_1, N_2 and N_3 remain constant in depth, is the effect of the angle transformation given by (2-2). As the sound speed increases away from the axis the angles c_S and c_B defining the vertical directionality are shifted towards zero as shown in Figure 5-1d. The results of this transformation are two-fold. First, the dominant RSR paths are given more weight by the cos θ-term in the calculation of omnidirectional noise:

$$N(z) = \int N_s(\theta;z) \cos\theta \, d\theta.$$

Second, the angular aperture of RSR paths, $\theta_B - \theta_S$, increases.



(a)



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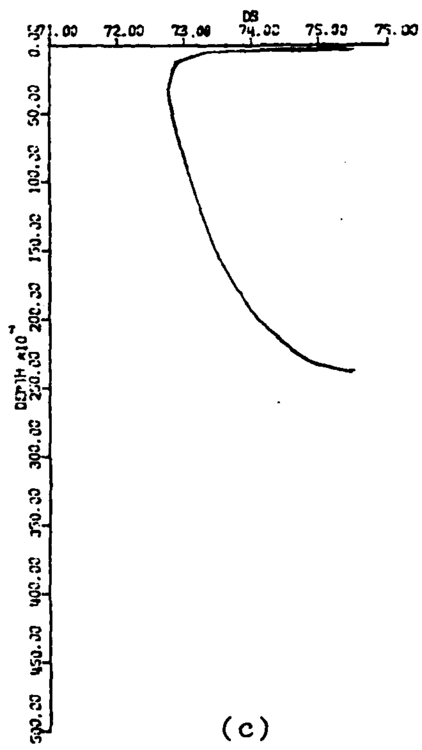
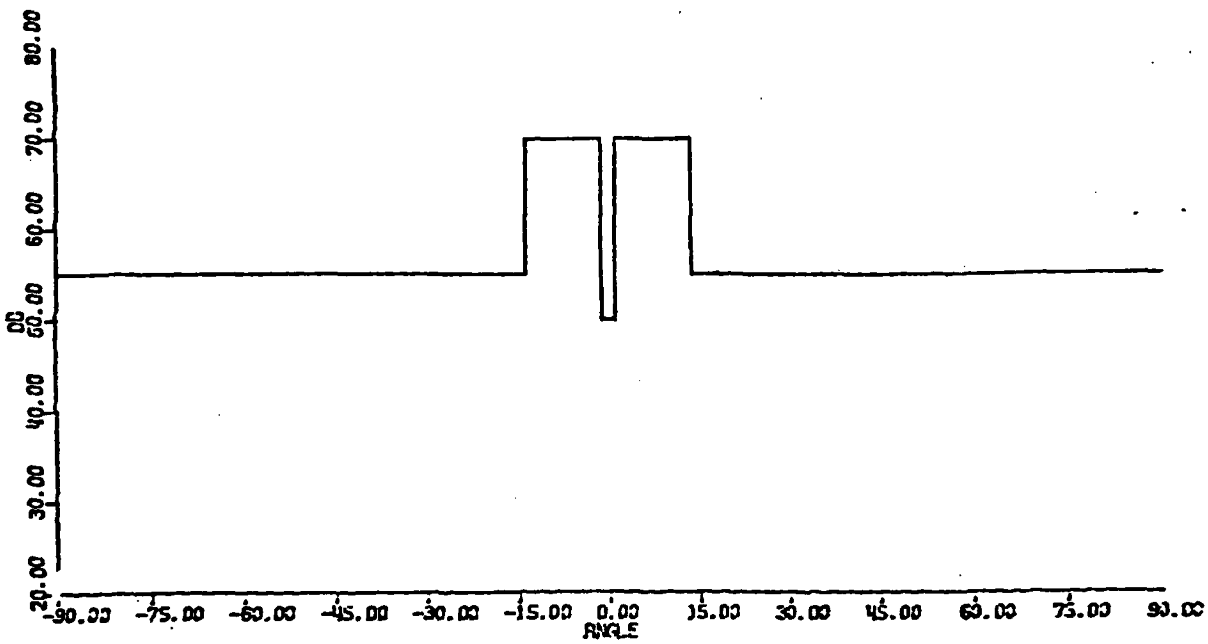


(c)

Figure 5-1. Environment One

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function



(d)

Figure 5-1. Environment One

(d) Vertical Directionality Near Critical Depth

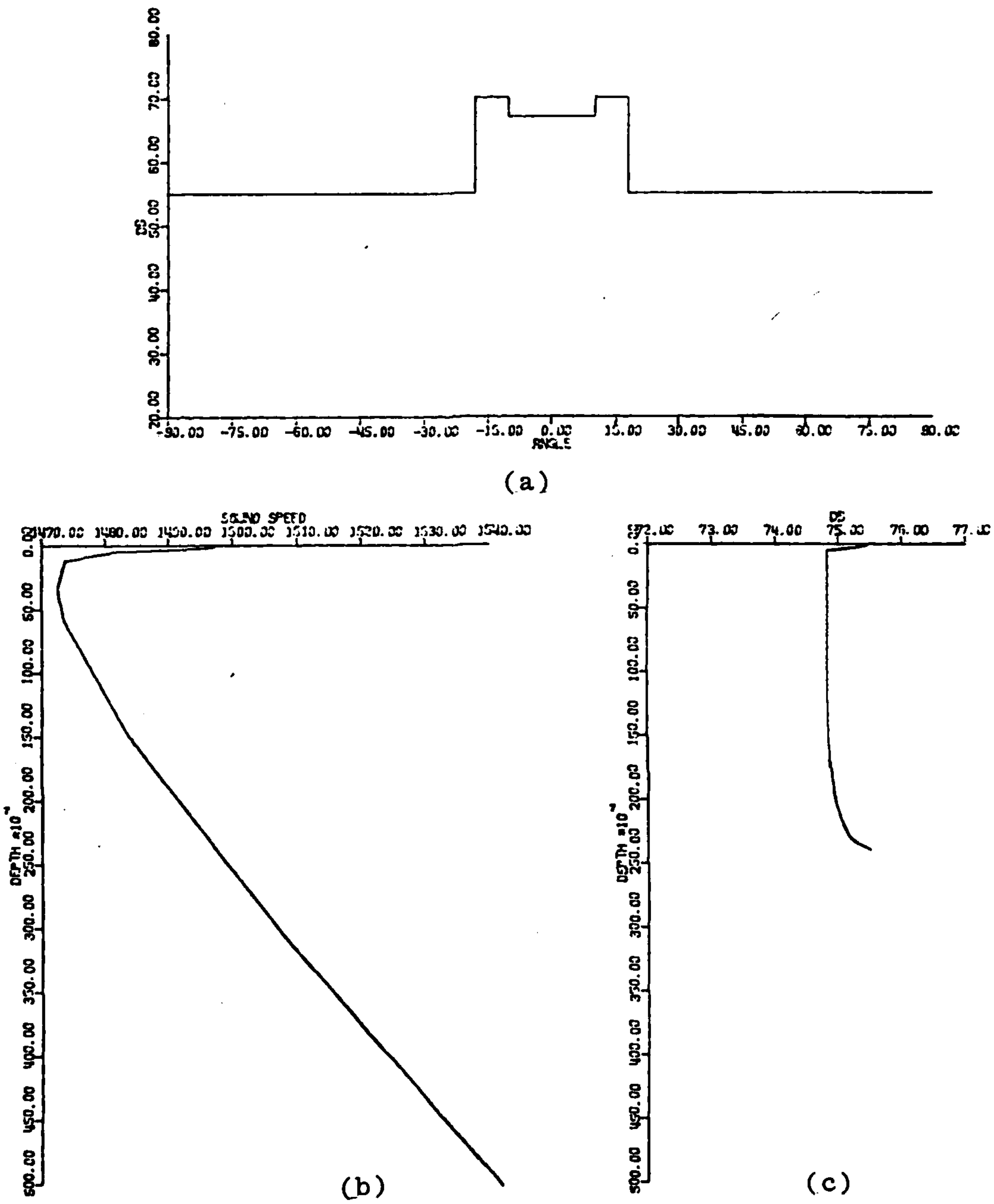
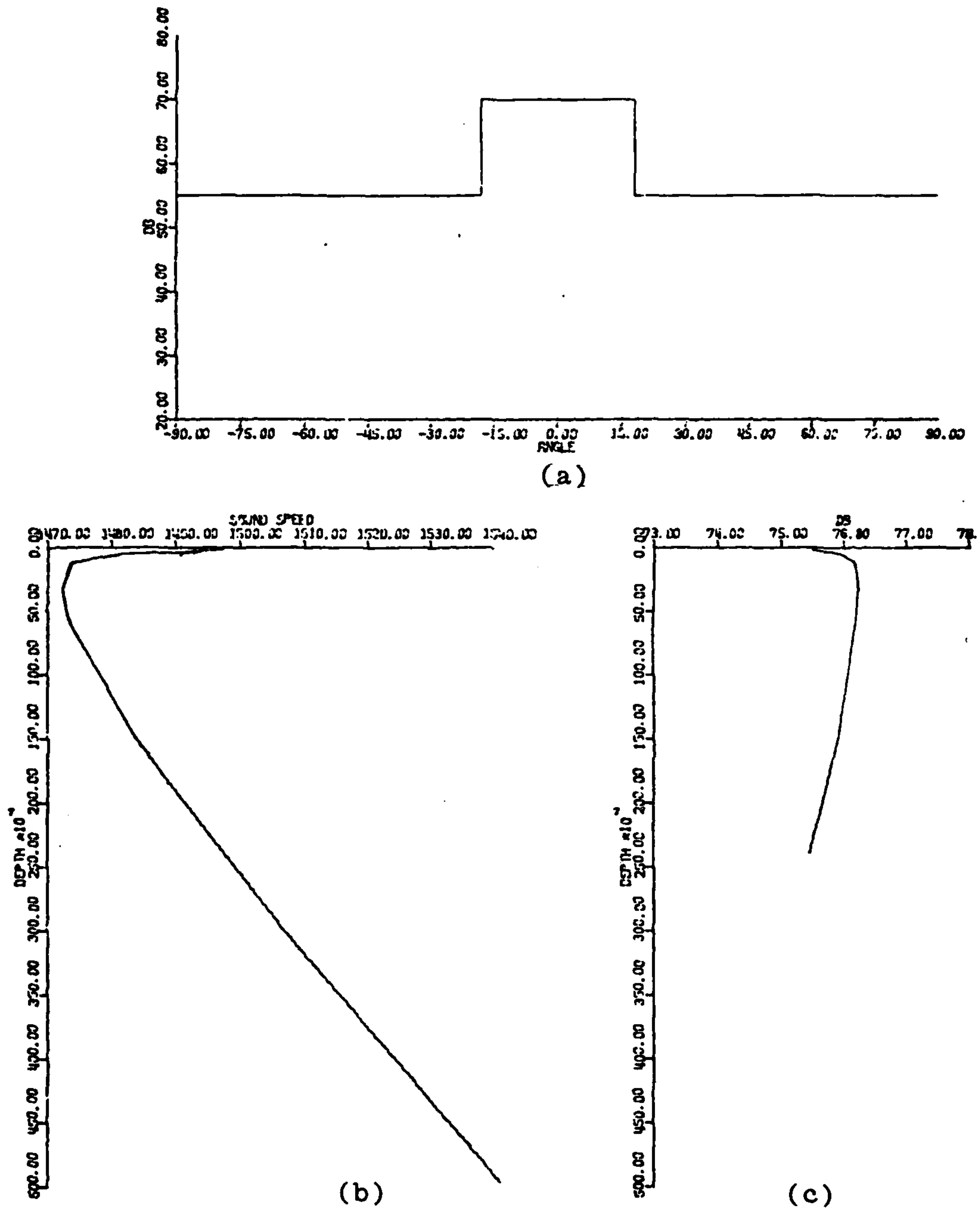


Figure 5-2. Environment One

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function



(a)

(b)

(c)

Figure 5-3. Environment One

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function

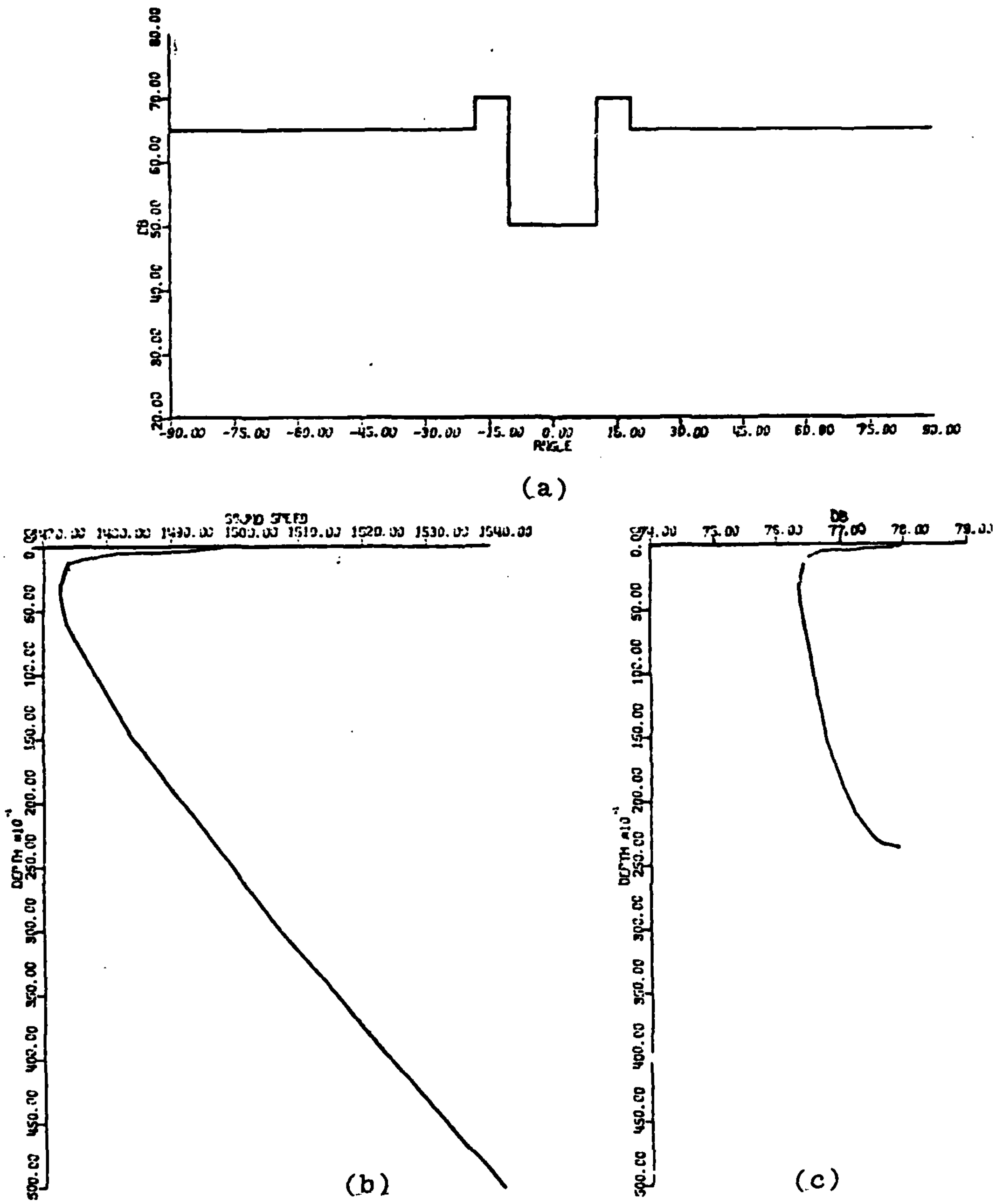


Figure 5-4. Environment One

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function

For this particular case, at the channel axis the RSR aperture is approximately 7.7° while at the critical depth it is approximately 14.8°. The second effect is generally an order of magnitude greater than the first.

Rather dramatic changes in the depth dependence occur as the noise level in the notch is increased. In Figure 5-2 where the notch level is 3 dB down from the RSR level, the depth-dependence function has become nearly constant. As the notch is filled further (Figure 5-3), the depth dependence becomes concave in the opposite direction so that it reaches a maximum at the sound-channel axis rather than a minimum as in Figure 5-1. This also is a result of Snell's Law. At depths where the sound speed is greater than at the axis, some of the shallower arrival angles in the notch are no longer seen. In Figure 5-1 the intensity of the arrival lost is negligible. However, as the notch is filled such energy becomes significant.

The final figure for this environment, Figure 5-4, demonstrates the damping effect on the depth-dependence function caused by the addition of more energy to the bottom-bounce paths. This is as expected since the energy in the RSR paths is then a smaller percentage of the total.

The second environment, Figures 5-5 thru 5-7, is an example of an extreme summer profile, with a strong thermocline and nearly bottom-limited conditions. At the channel axis, the surface and bottom grazing angles are approximately 15.4° and 16.6° respectively.

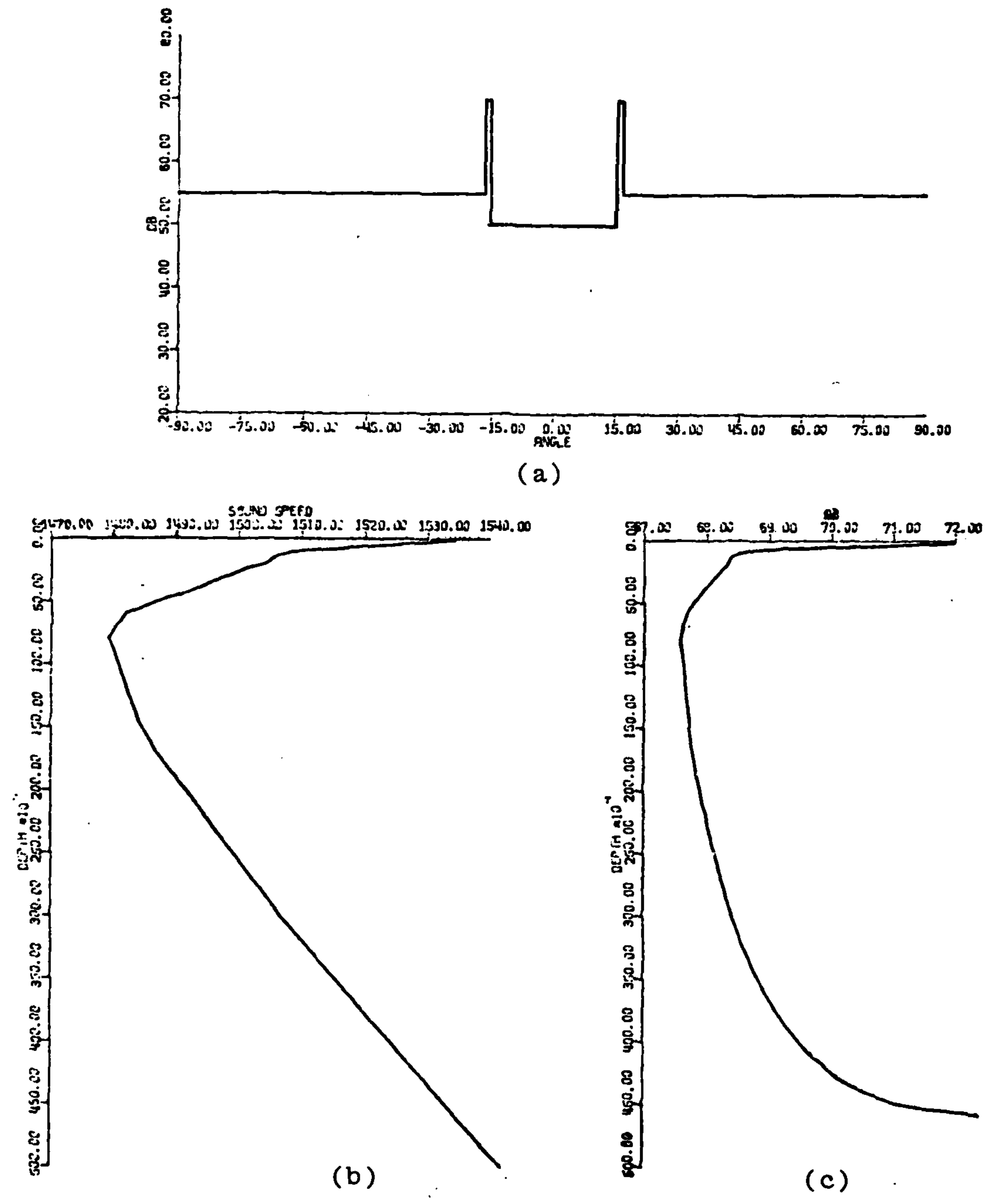
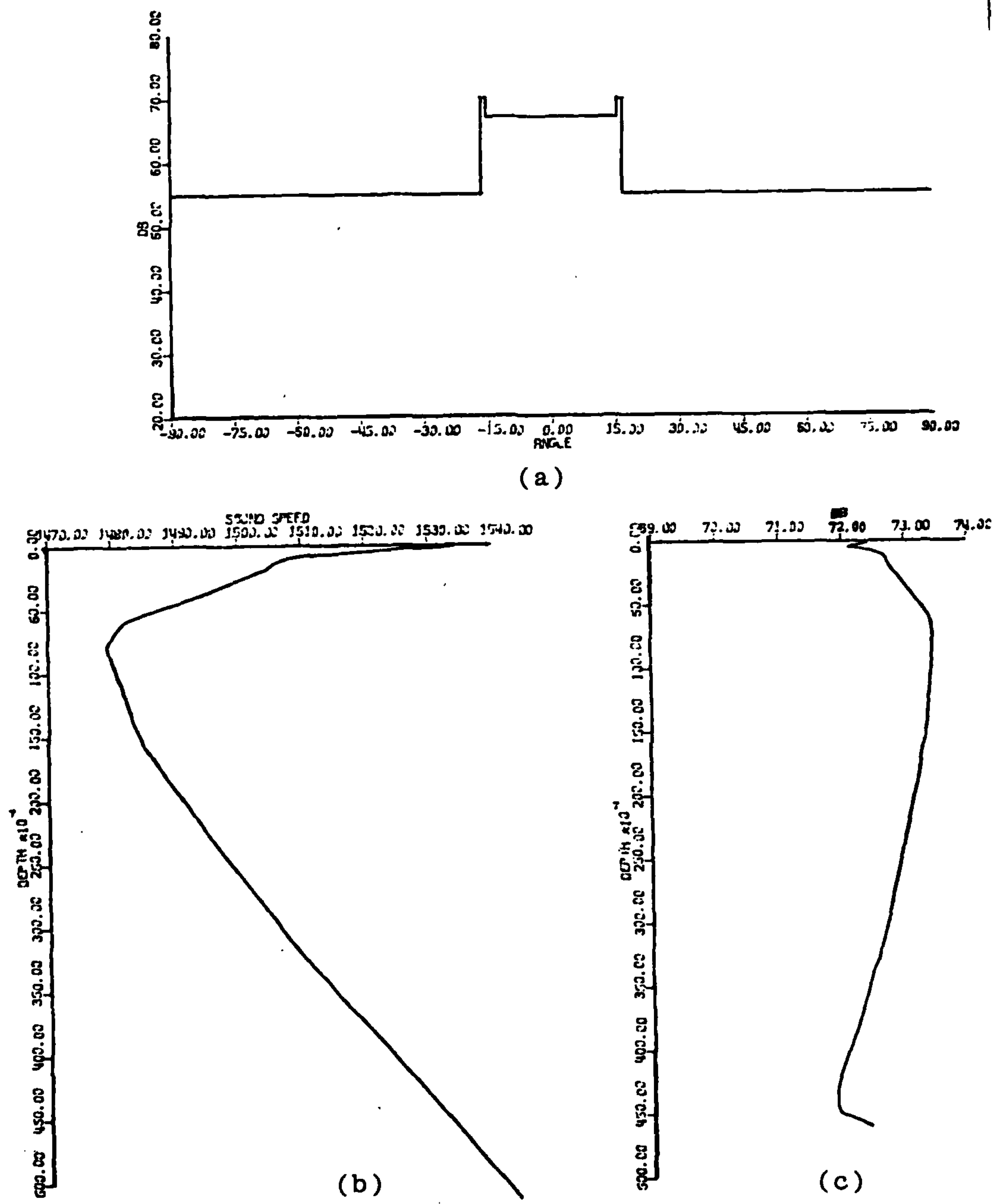


Figure 5-5. Environment Two

(a) Vertical Directionality at Channel Axis

(b) Sound Speed Profile

(c) Depth Dependence Function



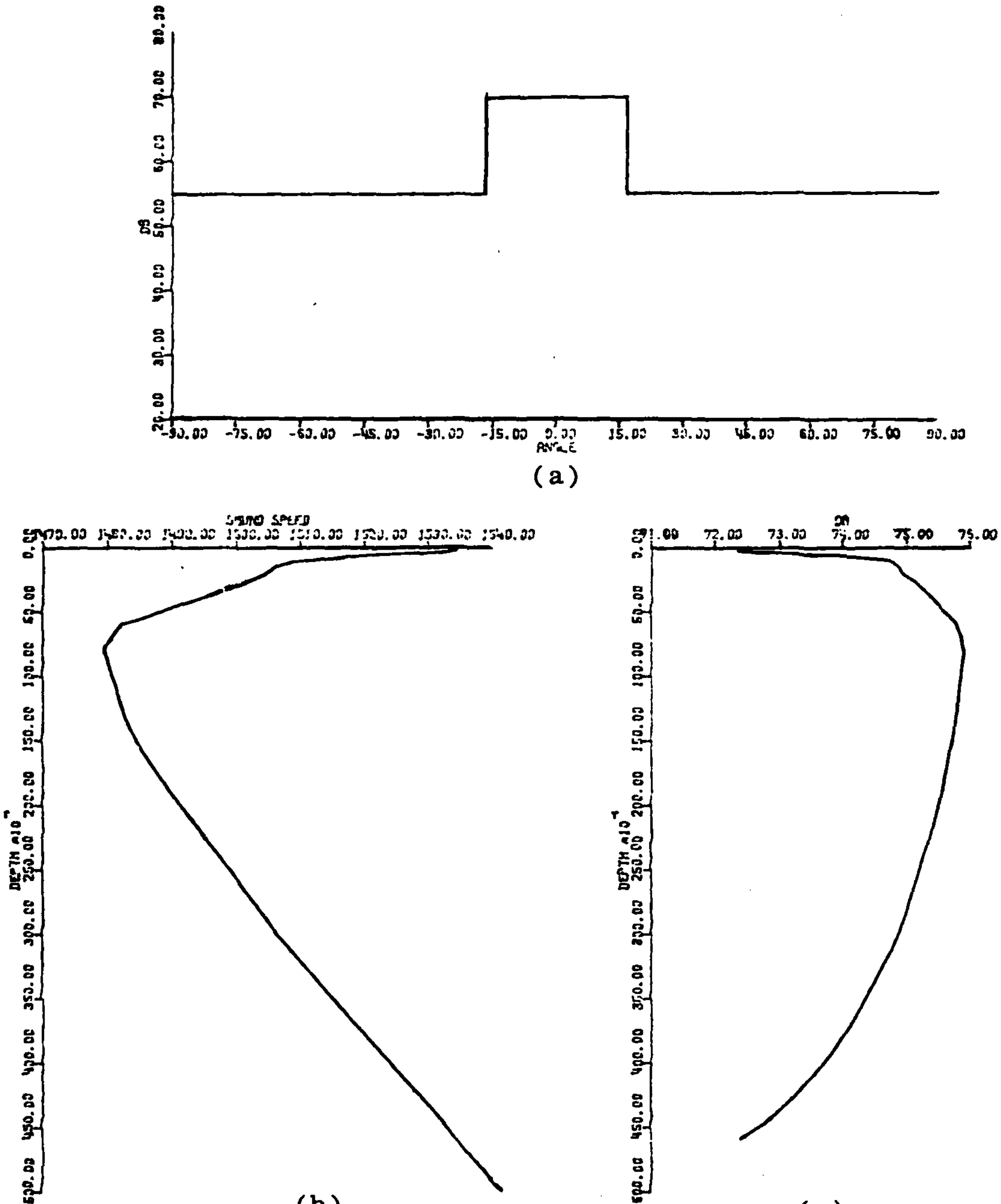
(a)

(b)

(c)

Figure 5-6. Environment Two

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function



(a)

(b)

(c)

Figure 5-7. Environment Two

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function

In comparison with Figure 5-1, the depth dependence function in Figure 5-5 has a range of nearly 5 dB. The increased variation is caused by the increase in the notch width and the narrowness of the RSR aperture at the channel axis. These two conditions result in the RSR aperture varying from 1.2° at the channel axis to 6.3° at critical depth.

Figures 5-6 and 5-7 again demonstrate that as the notch is filled the depth-dependence function completely changes shape. This change is more dramatic than in the first environment because proportionally more energy is being added.

The final environment, Figures 5-8 thru 5-10, further demonstrates the importance of the size of the RSR aperture. At the channel axis the surface grazing angle is 7.3°, smaller than either of the first two cases. Yet the depth dependence function in Figure 5-8 has a range of 3 dB, more than for the first environment. This happens because the width of the RSR aperture ranges from 1° to 4°, a variation which is proportionally greater than in the first case. The effect of filling the notch in this case, Figure 5-9 and 5-10, is similar to that of the previous example.

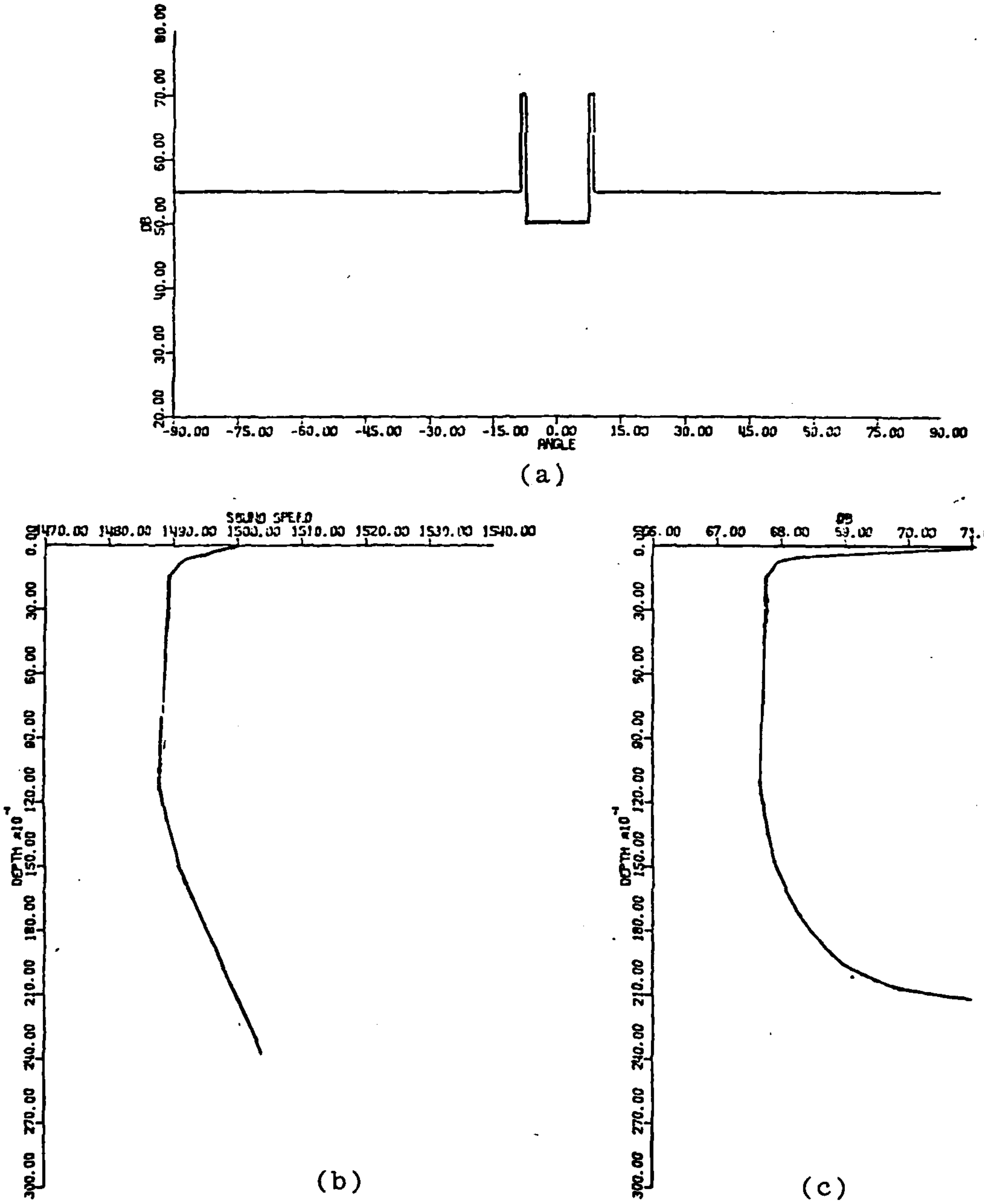


Figure 5-8. Environment Three

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function

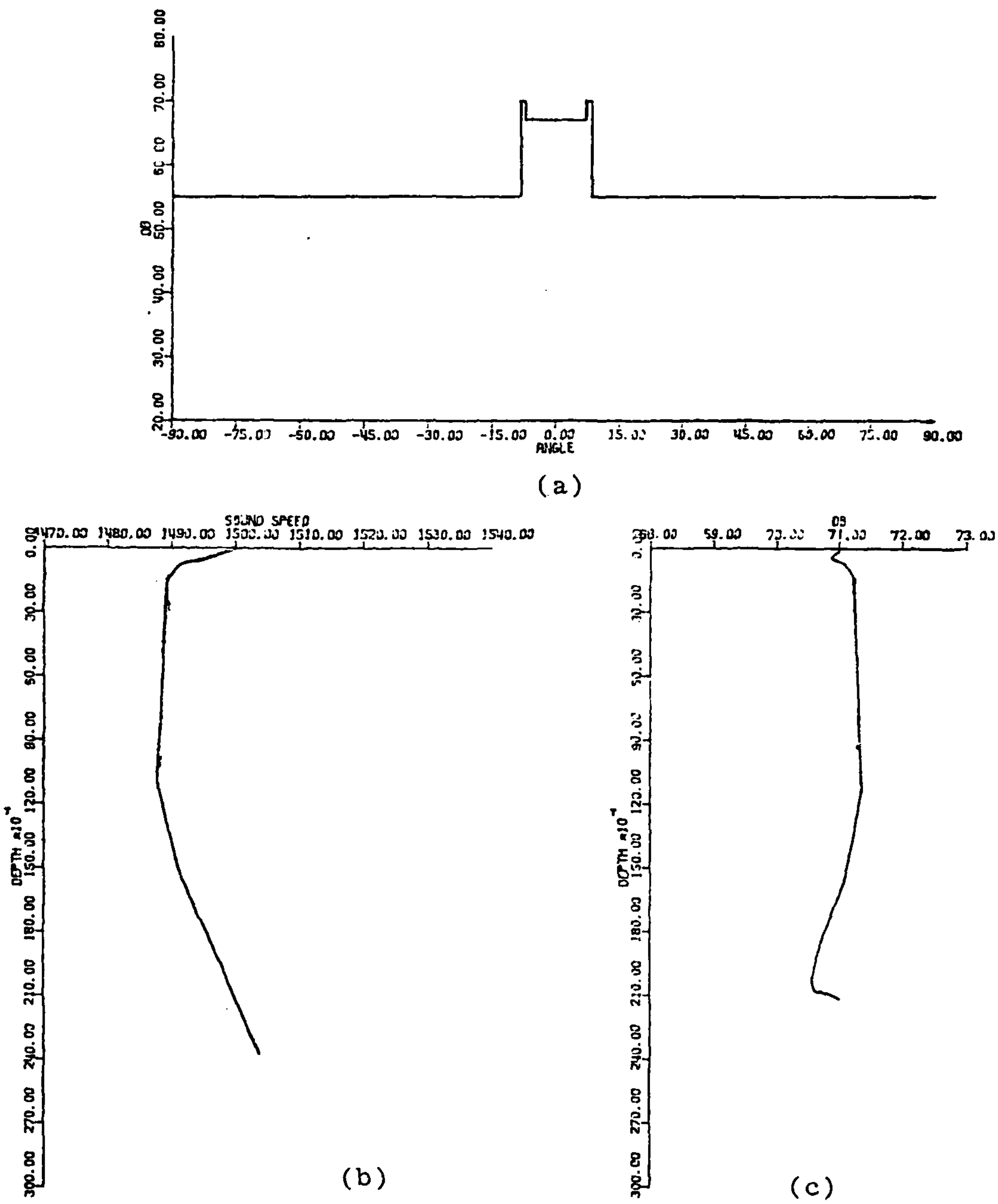


Figure 5-9. Environment Three

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function

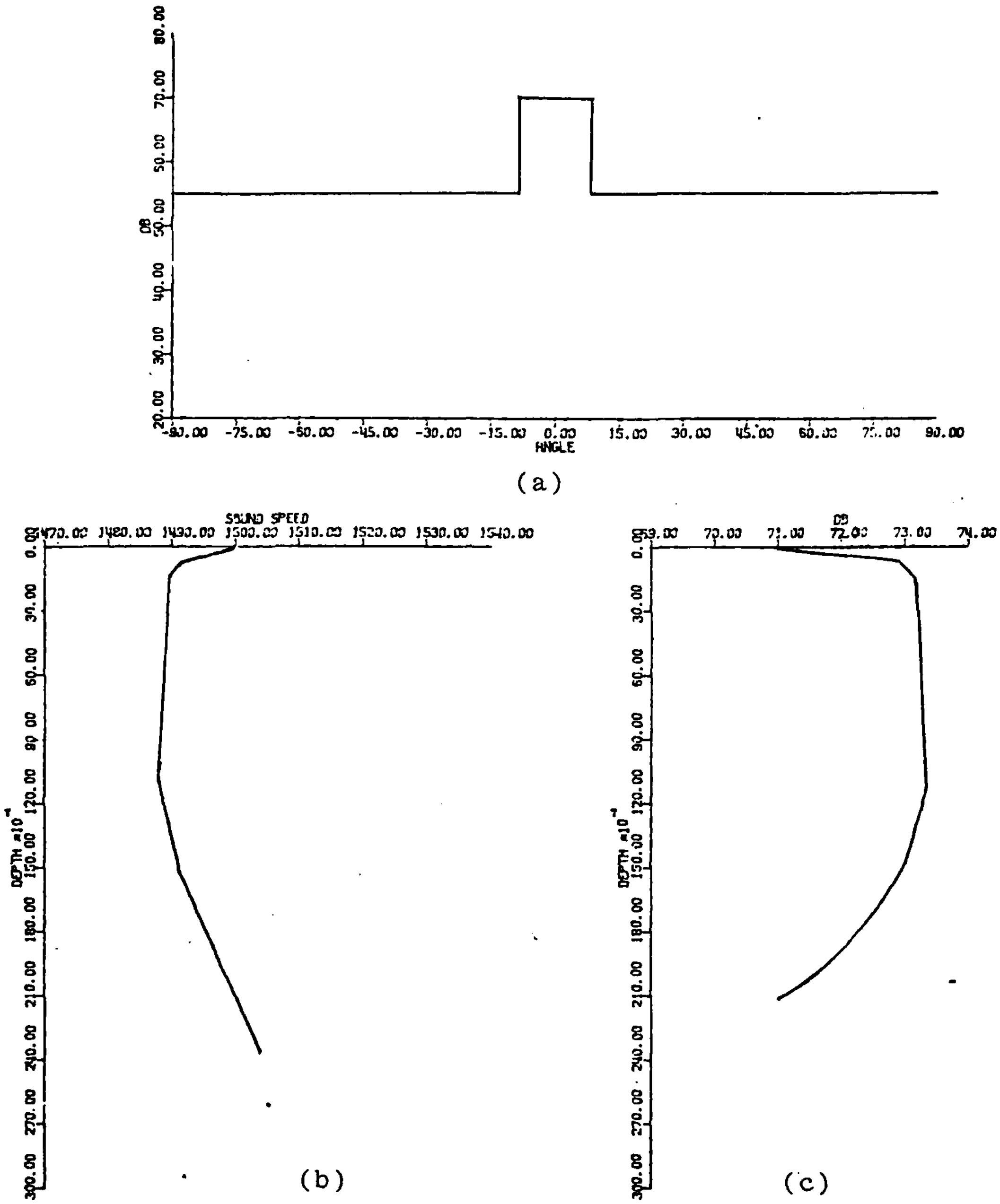


Figure 5-10. Environment Three

(a) Vertical Directionality at Channel Axis
(b) Sound Speed Profile
(c) Depth Dependence Function

In conclusion, these three examples suggest that the variability in the omnidirectional noise as a function of depth is dependent on:

(1) the width of the horizontal notch in the vertical directionality and the level to which it is filled,

(2) the width of the RSR aperture (particularly in the nearly bottom-limited cases),

(3) and, to a lesser extent, the amount of energy in the bottom-bounce paths.

Section 6
SUMMARY AND CONCLUSION

Beginning with a previously documented analytical result, a theoretical relationship between ambient noise vertical directionality at a single depth and the omni level as a function of depth has been derived and investigated. This earlier result was an approximation for noise vertical directionality as a function of depth based on a simple angle transformation. It allows omni level as a function of depth (restricted to depths within a single "sound channel ") to be estimated in terms of the vertical directionality at a single depth (the channel axis). The principal relationship, and those that follow from it, are limited by assumptions of range-averaged transmission loss and geometric acoustics. Extension to account for image interference and diffraction have been discussed.

Sections 3 and 5 described the specific effects that various cononical vertical directionalities have on the depth-dependence function, with closed-form expressions derived for special cases. Section 5 further investigates the effects of the environment for one of the canonical cases. Of particular importance is the relationship between the depth-dependence function and the amount of energy in the notch (the null near the horizontal predicted by range-independent noise models) in the vertical directionality. Examples show that the depth-dependence can be dramatically affected by energy added to the notch and to a slightly lesser extent by the width of the aperture of RSR arrivals.

The inverse relationship (directionality as a function of omni depth dependence) is established in Section 4 for symmetric directionality functions. A closed-form expression for vertical directionality is found for depth-dependence of a special form, and two error-minimization algorithms for approximating the vertical directionality are also described. Extension to the asymmetric case requires _a priori_ knowledge of the relationship in vertical directionality between positive and negative angles.

As discussed in Section 1, the motivation for this report was the desire to use existing depth-dependence data to provide a clue as to whether a notch at angles near the horizontal occurred in the vertical directionality. Unfortunately the data were too sparse (seldom more than two or three measurements in the main sound channel) for the results of Section 4 to yield more than coarse estimates. Search for better data and a definitive evaluation of the results are indicated.

Besides evaluation, future work should address:

- _a priori_ estimates of asymmetry based on bottom interaction,
- error minimization algorithms for canonical directionalities with singularities at the surface grazing angles,
- the impact of SII and wave effects on noise depth dependence,
- applications to transmission-loss models.

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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Underwater Acoustics, Range-Averaged Transmission Loss, Ambient Noise, Vertical Directionality, Depth Dependence, Ambient Noise Notch, Vertical Line Array

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The relationship between noise depth dependence and vertical directionality is considered in both directions. Under appropriate assumptions approximations for determining the depth-dependence from the vertical directionality at a specific depth are derived. Depth-dependence functions for several canonical directionality functions as well as specific examples is given. Two different approaches for determining the vertical directionality from the depth dependence are considered: an analytic

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20) solution is developed for the case that the depth dependence has a particular representation; an error-minimization algorithm is applied when the vertical directionality has a particular form.

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